
Maximum Flow Problem For Petroleum Product (Pms) Distribution System Using Ahmed Algorithm

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Abstract

In this work, maximum flow algorithm proposed by Ahmed et al (2014) was applied in the distribution of petroleum products through the NNPC pipelines from a source depot (Port Harcourt Refinery) to a sink depot (MaidugiriDepot). The pipeline network comprises of 8 depots (nodes) with 13 connecting routes (arcs). Each of these connecting pipes has a maximum quantity of product (in million litres) that can flow from one depot to the other at any particular supply. The Ahmed et al (2014) algorithm was then used to estimate the maximum flow. A total of 23 cuts were discovered and 7 augmenting paths were followed to achieve the maximum flow of 23 million litres after 7 iterations.

Keywords: *Maximum flow, Algorithm, Arc capacity, Augmenting path*

Introduction

The Maximum flow problem involves a directed network with arcs carrying flow. The only relevant parameter is the upper bound on arc flow, called arc capacity. The problem is to find the maximum flow that can be sent through the arcs of the network from some specified nodes, called the source, to a second specified node, called the sink. Originally the maximal flow problem was invented by Fulkerson and Dantzig in 1955 and solved by specializing the Simplex method for the linear programming, and Ford and Fulkerson solved it by augmenting path algorithm. The improvement of the Ford-Fulkerson method is Edmonds-Karp algorithm (1972) which performs better than the previous one. Jain and Garg (2012) proposed an improved version of Edmonds-Karp algorithm to solve the maximum flow problem, which requires less number of iterations and less augmentation to calculate the maximum flow. The algorithm is based on finding breakthrough paths with net positive flow between the source and sink nodes. Ahmed et al (2014) proposed an algorithm that calculates the maximum flow fast with lesser number of iterations than that which was proposed by Jain and Garg in 2012, hence the use of his algorithm for this work.

Applications of this maximum flow problem include finding the maximum flow of orders through a job shop, the maximum flow of water through a storm sewer system, and the maximum flow of product through a product distribution system, among others. In this work, it is applied in the petroleum product distribution through pipelines. In the case of petroleum distribution and transportation systems, two items play the key role in corresponding network structures: oil and gas. Whether the crude oil or gas are produced at domestic production units or imported through costal terminals from overseas, they should be transmitted through a nationwide distribution network to storage terminals or refinery centers. Then, they should be conveyed through pipelines, trucks, vessels, and other mechanisms to the final destinations which are primarily production plants, house users, etc. Apparently, such gigantic distribution and transportation system must deal with thousands of kilometers of pipelines (as the edges of networks) and hundreds of refinery centers and oil transition centers (as the vertices of networks); hence, one of the most primitive and fundamental objectives is to minimize the cost of flow as much as possible. Therefore, the techniques and mechanisms of minimum-cost flow problems can be implemented in order to secure this objective.

Purpose of study

To ascertain the maximum quantity of petroleum product to be transported from the source depot (Port Harcourt Refinery) to the destination (Maidugiri Depot).

Statement of problem

Whenever there is petroleum product from a source (Refinery) to be transported or transmitted to other destinations (Depot), there is need to minimize the cost of transportation/transmission and utilize the transportation/transmission medium so as to increase profit on sales. The problem becomes; what quantity of product is to be transported/transmitted from the source (Refinery) to the destination (Depot) and which shortest route to be followed in order to minimize costs and maximize profit. This work tends to use a proposed algorithm by Ahmed in 2014 totackle this problem. This will enable companies to take advantage of this opportunity to improve in their logistics.

Review of literature

Khan et al (2013) in their work introduced a new approach for finding the Maximum flow of a Maximal- flow problem which requires lesser number of iterations and less augmentation than Ford-Fulkerson algorithm using a numerical example is to illustrate the proposed method.

They also formulated the maximal-flow problem as a linear programming problem (LPP) and solved it using Bounded Variable Simplex Method. Ahmed et al (2014) introduced a new efficient algorithmic approach for finding the maximum flow of a maximal flow problem requiring lesser number of iterations and augmentation than Ford-Fulkerson algorithm. They provided a new algorithm for finding the amount of maximum flow from source(s) to sink(t) in a flow network Using numerical example to solve in order to illustrate the proposed algorithm and for the same problem the procedure at each iteration by using Ford-Fulkerson algorithm to compare their result in order to back up their argument. They went further to solve the same flow network problem by using bounded variable simplex method. At the end, their proposed algorithm returned the maximum flow that takes lesser number of iterations and augmentations than the Ford-Fulkerson Algorithm. Sangaiah et al (2014) try to investigate the Implementation of Dijkstra algorithm and Floyd algorithm in Microsoft Visual Studio in order to establish the storage structure of transportation network and find the shortest path between two cities. They formulated the optimal decision rules to analyze the time and cost parameters among various cities in china. Using Dijkstra and Floyd algorithm to compare the performance and provide optimal solution to the passengers. Jaafar et al (2014) opined that the maximum flow and minimum cost algorithm should not only be applied when products are involved but also in parking system. They suggested that parking system is one of the main important facilities that should have in any infrastructure or building especially for the place of interest and place of people's attraction. In their work, they applied Dijkstra's algorithm in finding the shortest path which helps in assigning the nearest vacant bay to drivers with necessary direction printed on the ticket so that drivers are able to find the 'best' lot with theminimum amount of time. Their work was achieved with the help of few electronic components such as PICs, IR sensors, push buttons, LEDs, LCDs, counters, comparators, and servo motors.

Methodology

The method applied in this work is the proposed algorithm by Ahmed et al (2014)

The proposed algorithm is given below:

Step 1: Initialize $f(u, v) = f(v, u) = 0$ for each edge $(u, v) \in E$

Where: u = Forward capacity, v = Backward capacity and E = Network capacity

Step 2: Set flow value =0

Step 3: If there exists any cut $[S, T]$ such that capacity (S, T) is equal to the flow value, then go to Step 8, otherwise go to step 4.

Step 4: Select an augmenting path P from source to sink in the residual network G_f .

Step 5: Set $C_f(P) = \min_{(u, v) \in P} \{C_f(u, v)\}$. Where $C_f(P)$ = Augmenting path capacities and

$\min_{(u, v) \in P} \{C_f(u, v)\}$ = the minimum capacity among the capacities on the augmenting path

Step 6: For eace $(u, v) \in P$, if $(u, v) \in E$, set $f(u, v) = f(u, v) + C_f(P)$ else $f(v, u) = f(v, u) - C_f(P)$.

Step 7: Set flow value = flow value + $C_f(P)$ and return to step 3.

Step 8: The maximum flow = flow value.

This algorithm is used to find the Maximum flow of petroleum product (PMS) from Port Harcourt refinery (Source) to Markurdi depot (Destination) through the pipelines. The pipeline network is giving below with the capacities (in hundred million litres) attached.

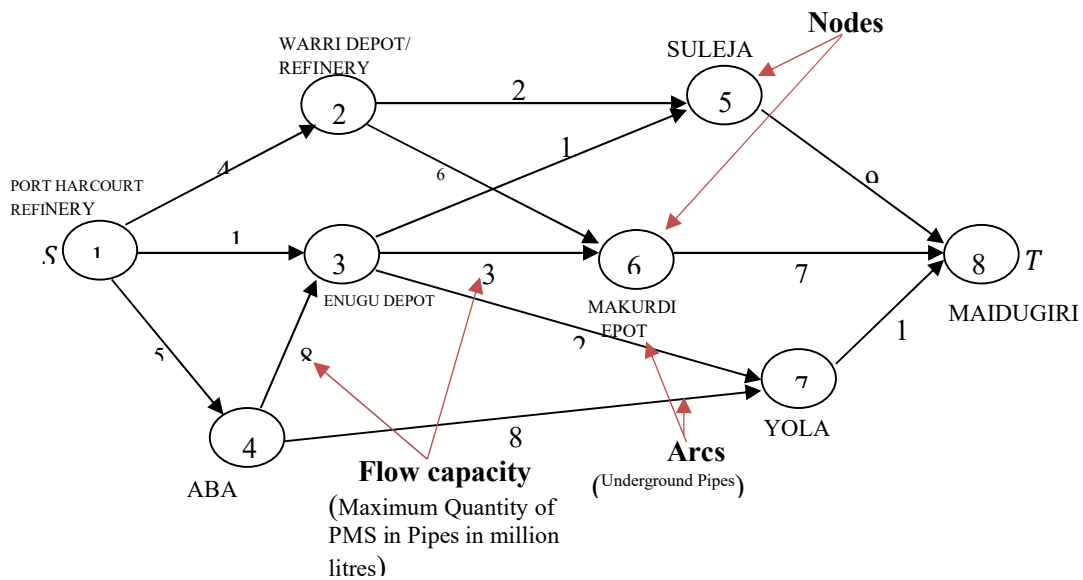


Figure 1: Pipeline network of PMS From Port Harcourt refinery to Maidugiri depot

The Maximum flow of the above network is then solved using the proposed algorithm of Ahmed et al (2014)

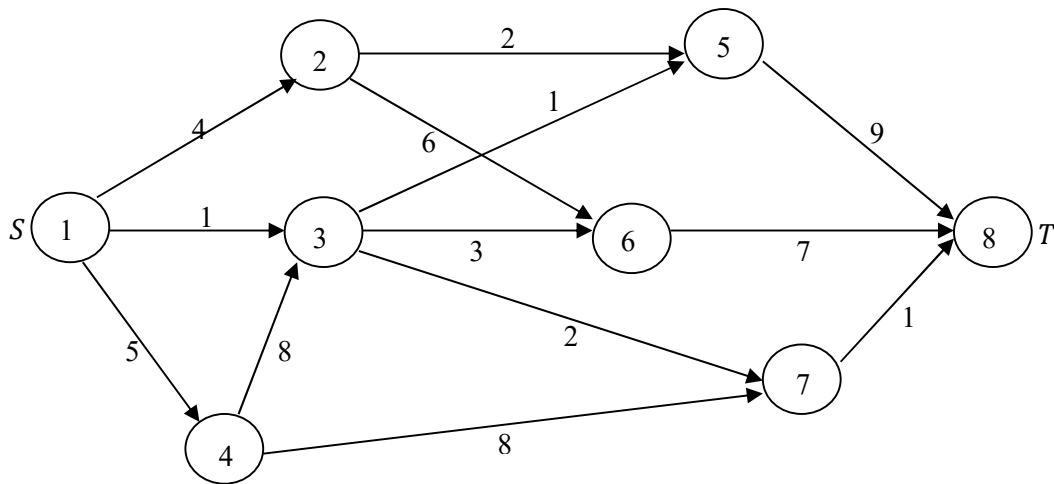


Figure 2: Considered network for solving the flow problem

Cuts

The figures 3 – figure 5 below show the different cuts found in the network. A cut could be defined as any disconnection that can prevent the flow from the source to the sink in a network. Here, each of the 21 different cuts will prevent the flow from source to sink in this network. Before we proceed, it will be of great importance to re-explain what a cut is. A cut is a set of arcs whose removal will interrupt all paths from the source to the sink. The capacity of a cut is the sum of the arc capacities of the set. The minimal cut is the cut with the smallest capacity. The algorithm described in this work solves only the maximum flow problem.

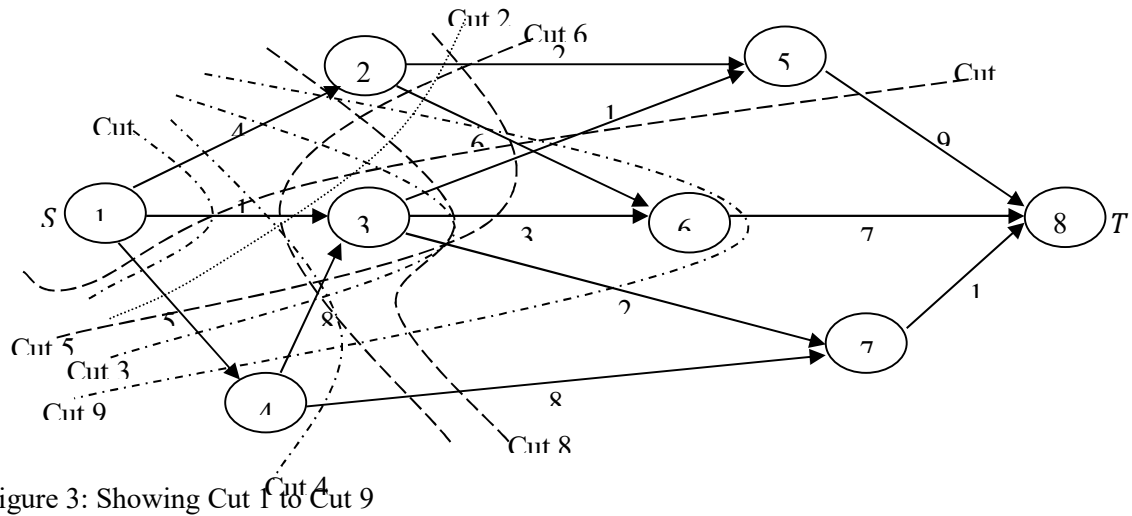


Figure 3: Showing Cut 1 to Cut 9

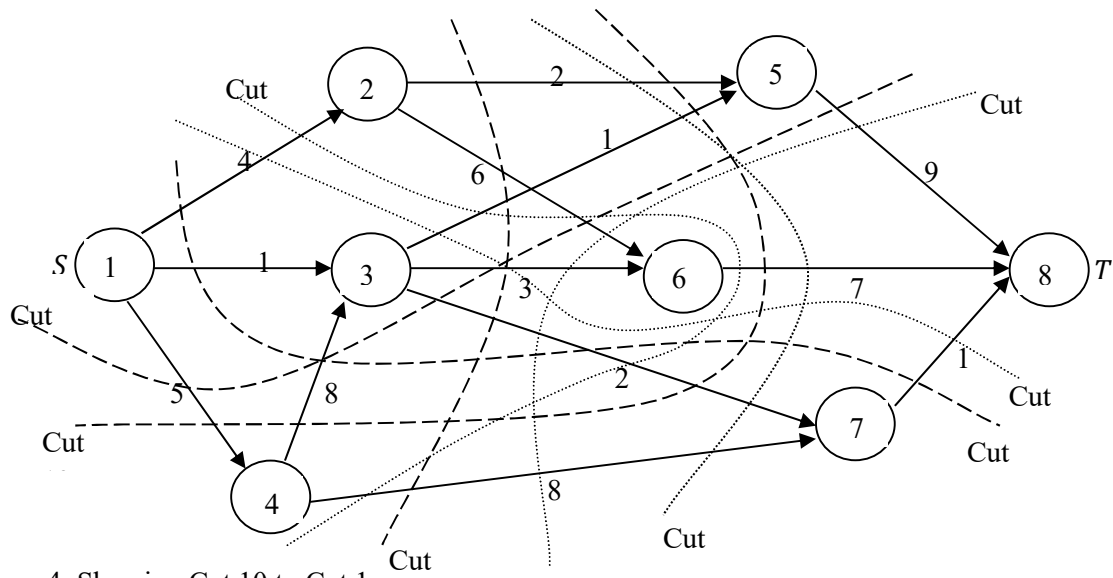


Figure 4: Showing Cut 10 to Cut 1,

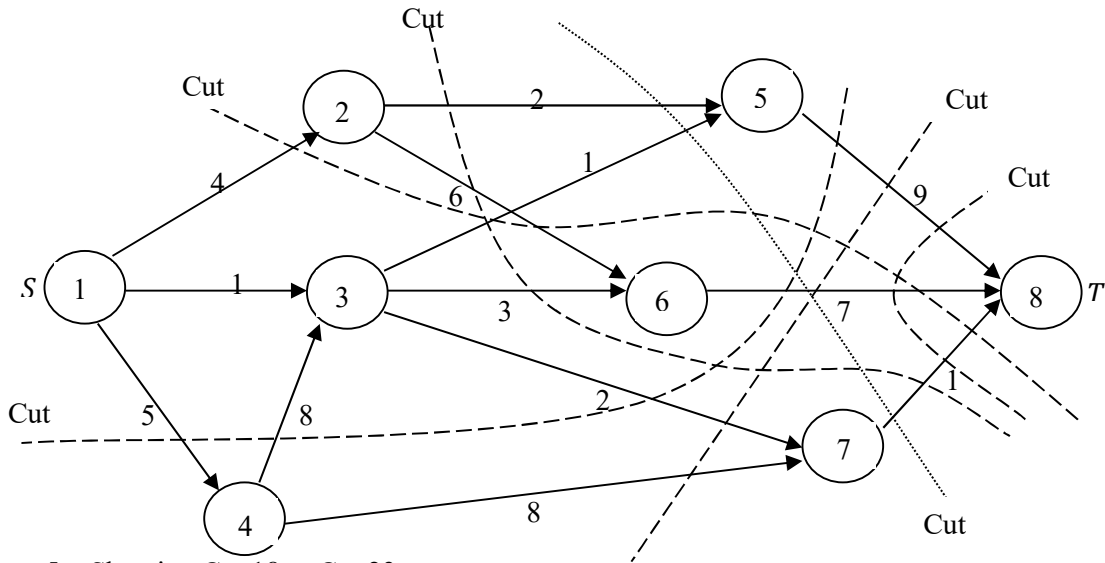


Figure 5: Showing Cut 18 to Cut 23

Data presentation and analysis

The data of this work was gotten from the 2021 NNPC bulletin. The researcher’s interest is to ascertain the maximum quantity that can be transmitted to Maidugiri depot and the shortest/best route to followed in order to minimize cost using Ahmed et al proposed algorithm. Source – Sink cut calculations. Here, the total capacity of the associated arcs that make up a cut identified in the above cut networks is calculated. The source – sink cut calculations shall be done from the figures 3.3 –5 above in the table below.

Table 1: Source-Sink cut calculation summary

Cuts	Source -Sink Cut	Affected Nodes	Associated Arcs	Cut Capacity	Total Capacity
1	1	2,3,4,5,6,7,8	(1-2),(1-3), (1-4)	4+18+5	27
2	1,2	3,4,5,6,7,8	(1-3),(1-4),(2-5),(2-6)	18+5+2+6	31
3	1,3	2,4,5,6,7,8	(1-2),(1-4),(3-5),(3-6),(3-7)	4+5+10+3+2	24
4	1,4	2,3,5,6,7,8	(1-2),(1-3),(4-3),(4-7)	4+18+8+8	38
5	1,2,3	4,5,6,7,8	(1-4),(2-5),(2-6),(3-5),(3-6),(3-7)	5+2+6+10+3+2	28
6	1,2,4	3,5,6,7,8	(1-3),(2-5),(2-6),(4-3),(4-7)	18+2+6+8+8	42
7	1,2,5	3,4,6,7,8	(1-3),(1-4),(2-6),(5-8)	18+5+6+9	38
8	1,3,4	2,5,6,7,8	(1-2),(3-5),(3-6),(3-7),(4-7)	4+10+3+2+8	30
9	1,3,6	2,4,5,7,8	(1-2), (1-4),(3-5),(3-7),(6-8)	4+5+10+2+7	28
10	1,4,7	2,3,5,6,8	(1-2),(1-3),(4-3),(7-8)	4+18+8+10	40
11	1,2,3,4	5,6,7,8	(2-5),(2-6),(3-5),(3-6),(3-7),(4-7)	2+6+10+3+2+8	31
12	1,2,3,5	4,6,7,8	(1-4)(2-6),(3-6),(3-7),(5-8)	5+6+3+2+9	25
13	1,2,3,6	4,5,7,8	(1-4),(2-5),(3-5),(3-7),(6-8)	5+2+10+2+7	26
14	1,3,4,6	2,5,7,8	(1-2),(3-5),(3-7),(4-7),(6-8)	4+10+2+8+7	31
15	1,3,4,7	2,5,6,8	(1-2),(3-5),(3-6),(7-8)	4+10+3+10	27
16	1,2,3,4,	6,7,8	(2-6),(3-6),(3-7),(4-7),(5-8)	6+3+2+8+9	28

	5				
17	1,2,3,4, 6	5,7,8	(2-5),(3-5),(3-7),(4-7),(6-8)	2+10+2+8+7	29
18	1,2,3,4, 7	5,6,8	(2-5),(2-6),(3-5),(3-6),(7-8)	2+6+10+3+10	31
19	1,2,3,5, 6	4,7,8	(1-4),(3-7),(5-8),(6-8)	5+2+9+7	23
20	1,3,4,6, 7	2,5,8	(1-2),(3-5),(6-8),(7-8)	4+10+7+10	31
21	1,2,3,4, 5,6	7,8	(3-7),(4-7),(5-8),(6-8)	2+8+9+7	26
22	1,2,3,4, 6,7	5,8	(2-5),(3-5),(6-8),(7-8)	2+10+7+10	29
23	1,2,3,4, 5,6,7	8	(5-8),(6-8),(7-8)	9+7+10	26

Calculation of maximum flow by Ahmed proposed Algorithm

To get the maximum flow, we shall consider each of the augmented paths starting from 1 to 6 as shown in the table below and other paths that will be established in the course of determining the maximum flow from the already established paths. The table below gives the summary of the paths.

Table 2: Source-Sink path summary

Paths	Associated Arcs
1	1 → 2 → 5 → 8
2	1 → 2 → 6 → 8
3	1 → 3 → 5 → 8
4	1 → 3 → 6 → 8
5	1 → 3 → 7 → 8
6	1 → 0 → 7 → 8

Using the proposed algorithm of Ahmed et al (2014);

Initialization: Set the value of the given capacities in the network to zero (0) as shown below

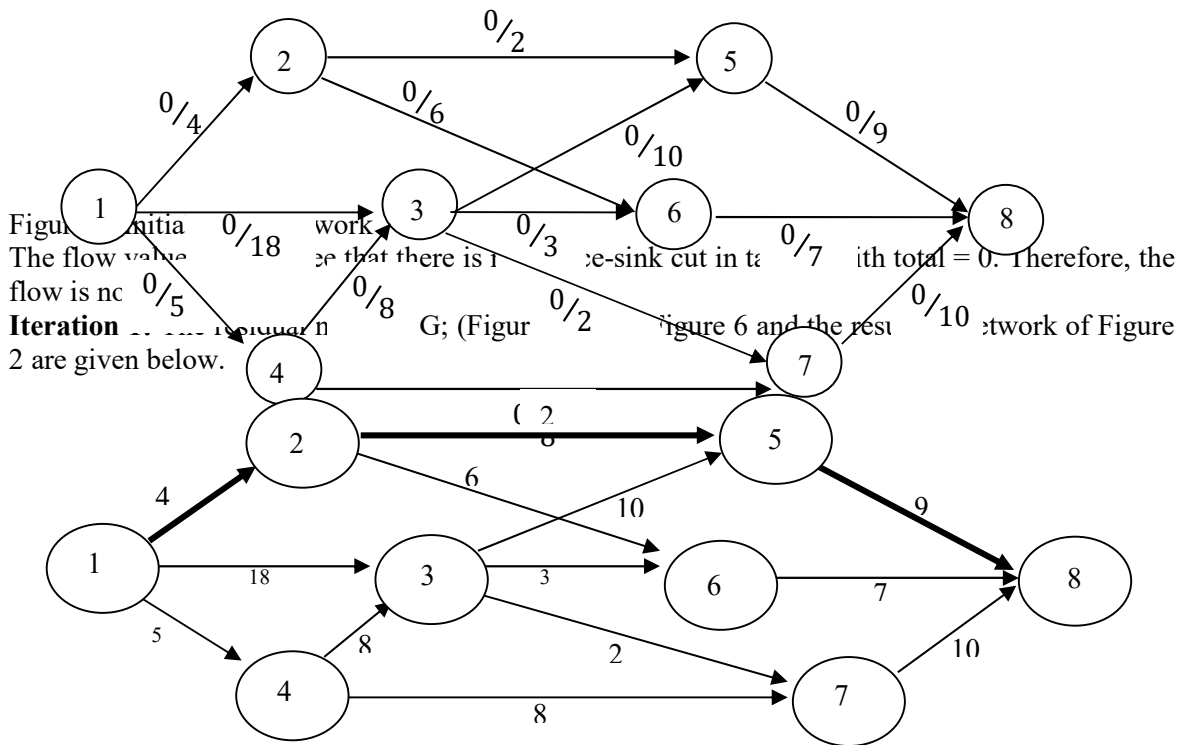


Figure 7: Residual network of iteration 1

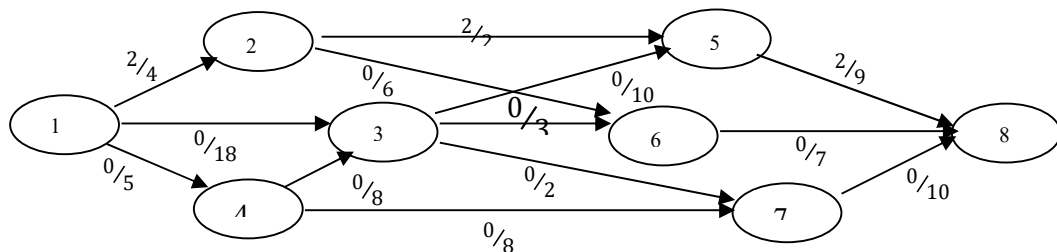


Figure 8: Allocation network of iteration 1

In the above case, from the path $1 \rightarrow 2 \rightarrow 5 \rightarrow 8$ by $C_f(p) = \min(4, 2, 9) = 2$ and so, the flow value $= 0 + C_f(p)$ is $0 + 2 = 2$. The value 2 is not found in table 1 above at total capacity column, so the flow is not at maximum.

Iteration 2: Consider the path $1 \rightarrow 2 \rightarrow 6 \rightarrow 8$

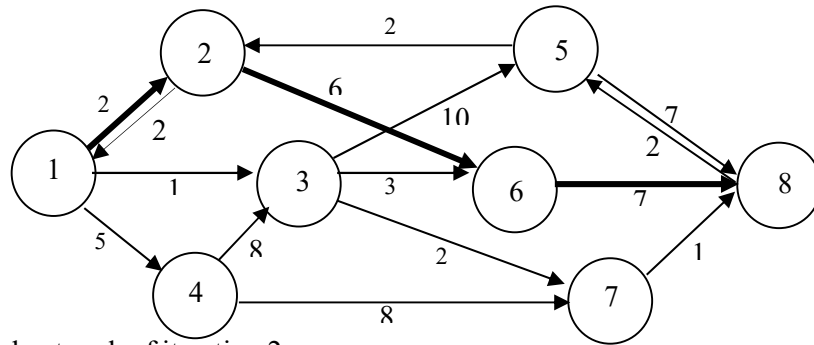


Figure 9: Residual network of iteration 2

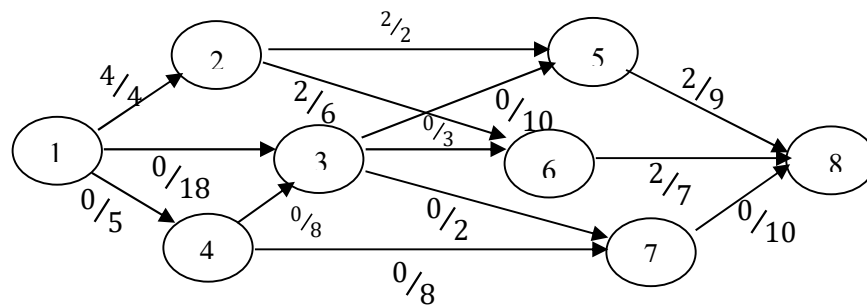


Figure 10: Allocation network of iteration 2

In this case, $C_f(p) = \min(2, 6, 7) = 2$, the flow value = $2 + C_f(p) = 2 + 2 = 4$. But 4 is not in the total column of table 1, so the flow is not at maximum.

Iteration 3: consider the path $1 \rightarrow 3 \rightarrow 5 \rightarrow 8$ in the residual network G of figure 9 which is given in figure 11 and the resulting network in figure 12 below.

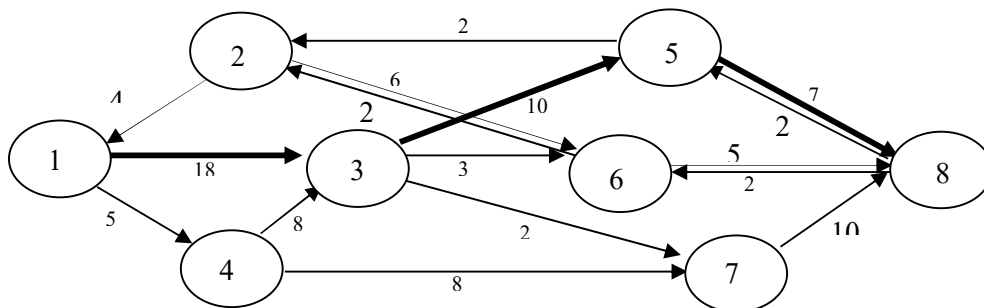


Figure 11: Residual network of iteration 3

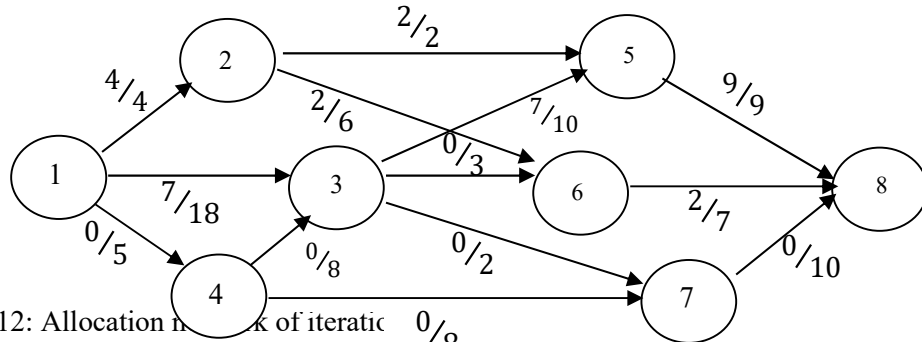


Figure 12: Allocation network of iteration 3
 Here, we can see that $C_f(p) = \min(18, 10, 8) = 7$, then the flow value = $4 + C_f(p) = 4 + 7 = 11$. Since the flow value 11 is not found in table 1 above, then, the flow is not at its maximum.

Iteration 4: Considering the path $1 \rightarrow 3 \rightarrow 6 \rightarrow 8$ the residual network of figure 11 is given in figure 13 and the resulting network below

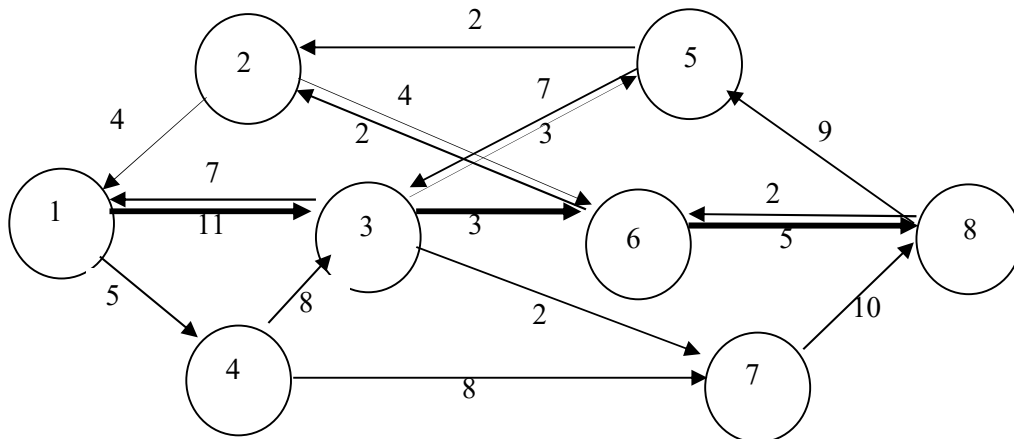


Figure 13: Residual network of iteration 4

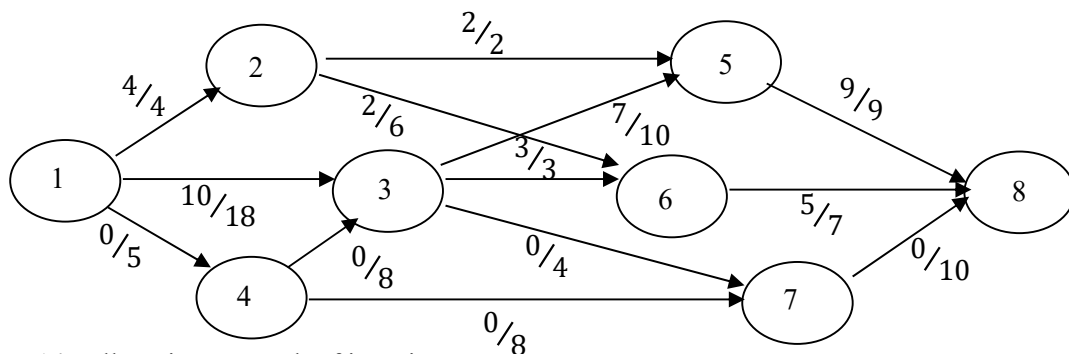


Figure 14: Allocation network of iteration 4
 Here, we can see that $C_f(p) = \min(11, 3, 5) = 3$, then the flow value = $11 + C_f(p) = 11 + 3 = 14$ which is not found in the total capacity column of table 1. So, the flow value is not at its maximum.

Iteration 5: We then consider path $1 \rightarrow 3 \rightarrow 7 \rightarrow 8$ the residual network of figure 13 is given in figure 15 and the resulting network in

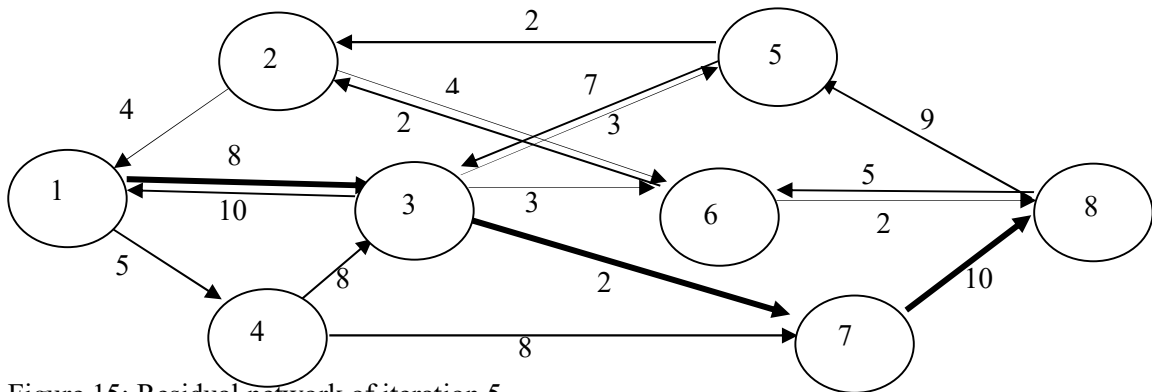


Figure 15: Residual network of iteration 5

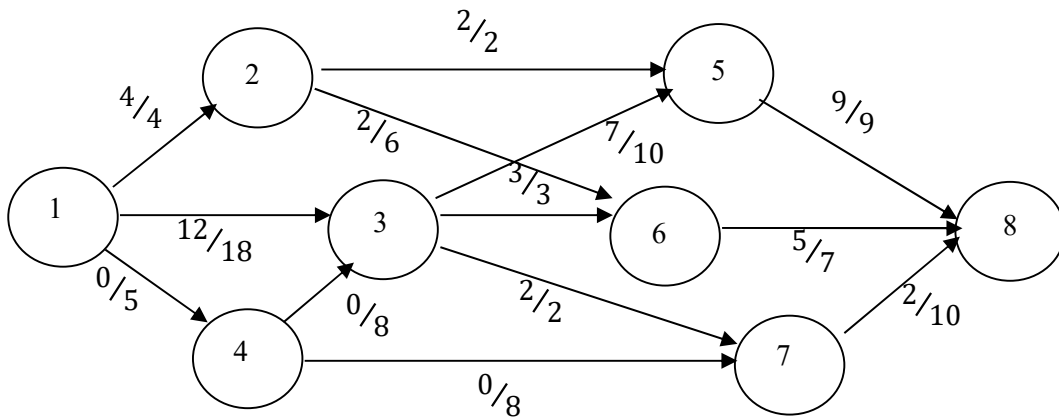


Figure 16: Allocation network of

In this case, $C_f(p) = \min(8, 2, 10) = 2$, then the flow value = $14 + C_f(p) = 14 + 2 = 16$ which does not exist in the source-sink of table. So, the flow value is not at its maximum.

Iteration 6: We then consider path $1 \rightarrow 4 \rightarrow 7 \rightarrow 8$ the residual network of figure 15 is given in figure 17 and the resulting network in figure 18 is

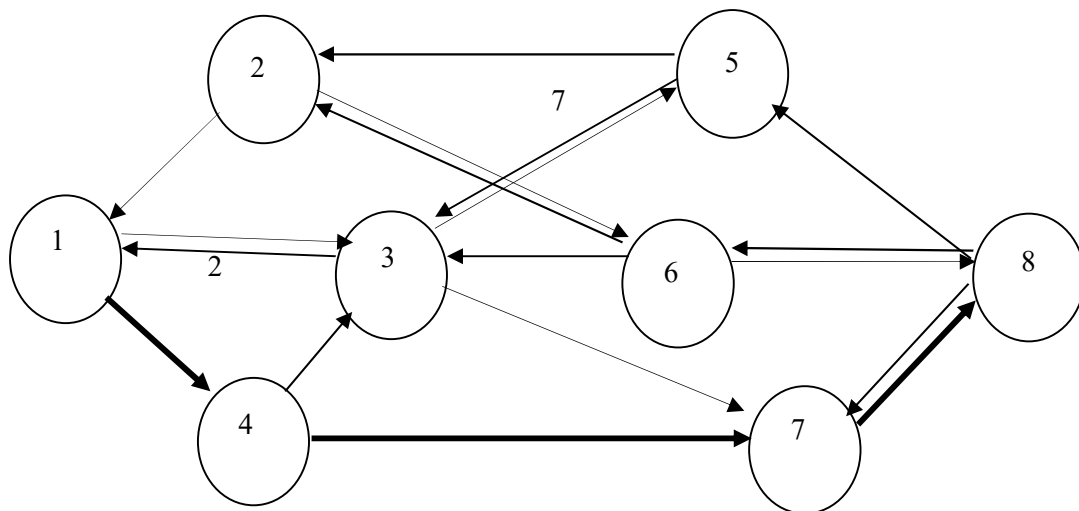


Figure 17: Residual network of iteration 6

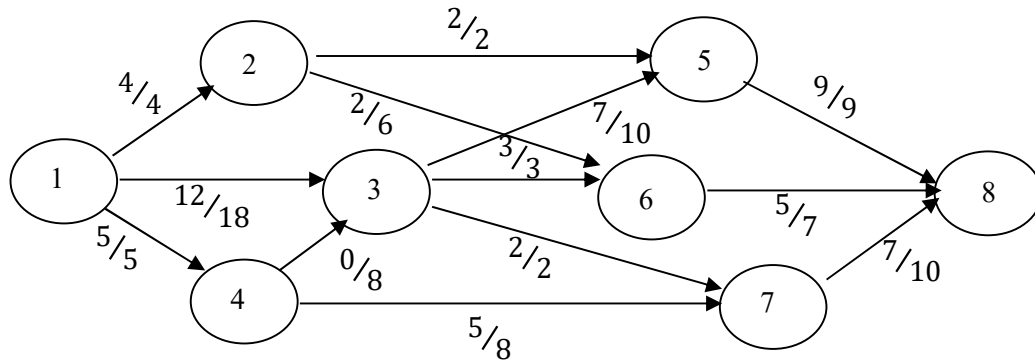
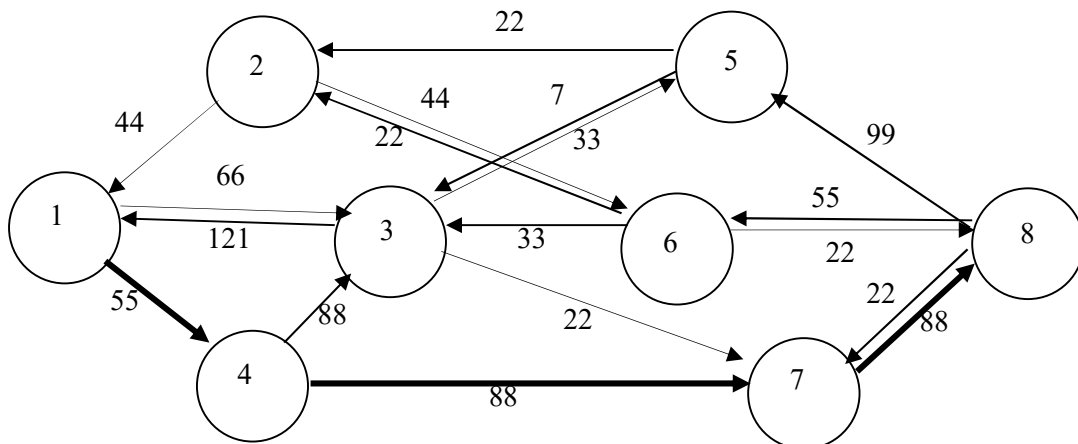


Figure 18: Allocation network of iteration

Here, we can see that $C_f(p) = \min(5, 8, 8) = 5$. The flow value = $16 + C_f(p) = 16 + 5 = 21$. Since the flow value 21 is not found in table 1 above, then, the flow is not at its maximum.



Iteration 7: Looking at figure 17, we can see that the only movement from the source that can connect to the sink is $1 \rightarrow 3 \rightarrow 5 \rightarrow 2 \rightarrow 6 \rightarrow 8$ this is the only possible path from source to sink. The residual network from figure 17 is given as seen in figure 19 below.

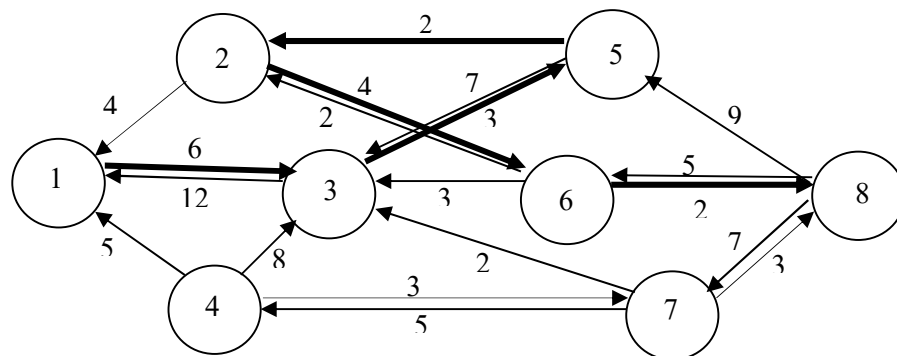


Figure 19: Residual network of iteration 7

Considering the path

$$\text{the } C_f(p) = \min(6, 3, 2, 4, 2) = 2.$$

Then the flow value becomes $21 + C_f(p) = 21 + 2 = 23$. Since there is no possible path from the source to the sink also, the value 23 is found in the source-sink cut of table 1 at cut 19 with source-sink (1, 2, 3, 5, 6), we proceed to the capacity network below:

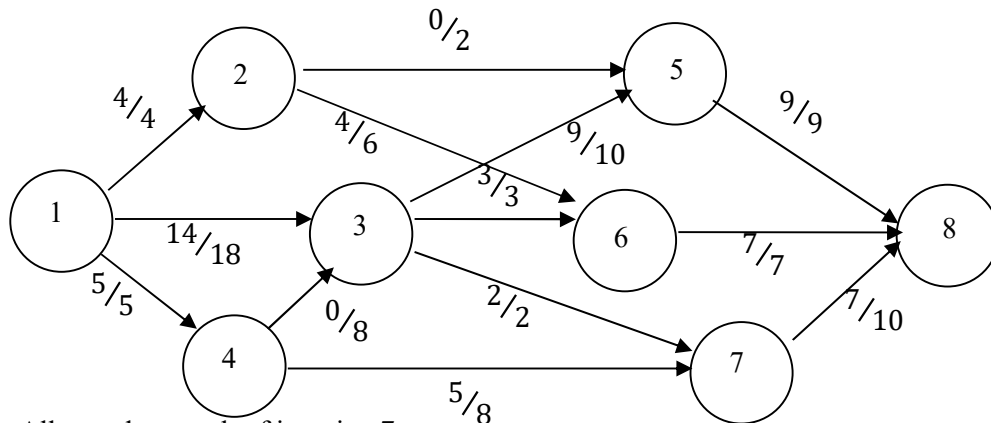


Figure 20: Allocated network of iteration 7

Since we have gotten flow value as stated, the algorithm is terminated at **iteration 7** and is considered as the maximum flow with value 23.

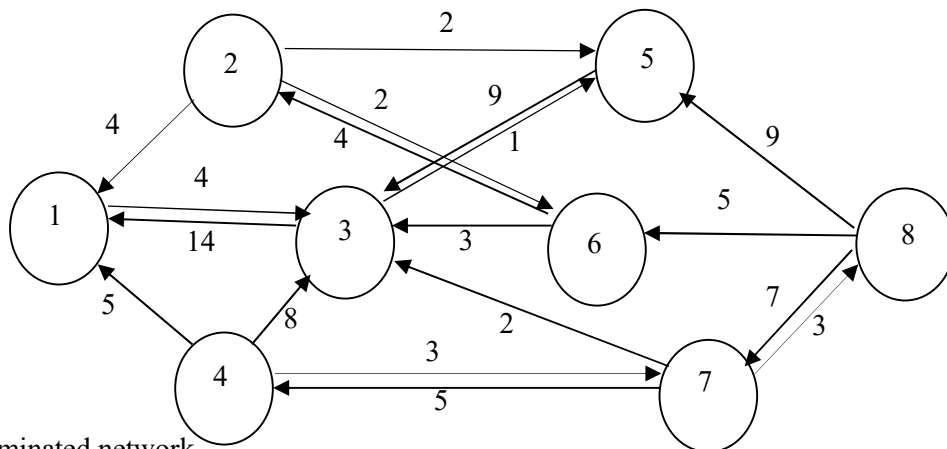


Figure 21: Terminated network

There is no possible flow in the network below from the source to the sink. this shows that the network's flow has terminated and the total flow values from all the iterations involved made up the maximum flow value.

Conclusion

It was observed from the pipeline network that transporting premium motor spirit from the Port Harcourt refinery to the Maidugiri depot, a total of 23 cuts capable of distorting or preventing the flow of the said product from its source to its sink is involved in this network. The maximum quantity of product to flow through the pipeline from the source to the sink using the proposed algorithm of Ahmed et al (2014) is 23 million litres.

Recommendation(s)

From the work, we conclude that the augmenting path from the source (Port Harcourt Refinery) to the sink (Maidugiri depot) to achieve the maximum flow of the 23 million litres recorded from the network is from Port Harcourt refinery to Enugu depot to Suleja depot to Warri depot back to Enugu depot then proceed to Makurdi depot and finally to Maidugiridepot which is the sink (final destination). However, it is important to embark on further research from this work to ascertain: (i) the circumference of the underground pipe (ii) the cost implications and maximum profit of the product after sales as these were not done in this work.

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