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## Application Of Markov Model To The Study Of Breast Cancer: A Case Study Of Patients In University Of Port Harcourt Teaching Hospital

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### **Abstract**

*The paper studied the problem of Breast cancer in Rivers State using Markov Chain Model. The data for this study was obtained from Cancer Registry Unit; University of Port Harcourt Teaching Hospital. These sample data were transformed to  $3 \times 3$  matrix, otherwise known as transition count matrix, denoted as  $M$  and it's corresponding probability transition matrix denoted by  $P$ . Markov Model was used to determine the ages of women that are susceptible to breast cancer and the number of women who transited from one state to another, like-wise those that left the system as a result of death. The plot of the raw data against time showed that as the years go by, the number of breast cancer patients increased. This study also proffered solution to factors that could reduce the risk of breast cancer.*

**Keywords:** *Markov Chain, Breast cancer, Rivers State and Transition Matrix.*

## Introduction

Breast cancer is the abnormal, uncontrolled proliferation of cells originating from the breast tissue. It is more common for breast cancers to emanate from the lining of the milk ducts or the lobules supplying the ducts (Brinton et al; 2014). It is the most common cause of cancer in women and also the leading cause of cancer death among women, especially in less developed countries. However, in more developed countries like the United States, lung cancer is the leading cause of death among women followed by breast cancer (Cancer facts; 2009). It is estimated that over 1 million new cases of breast are diagnosed every year constituting serious health threat to women all over the world (Colditz & Roster; 2000). Prevalence of breast cancer varies with different regions, hence the Western countries are said to have higher number of cases compared with their African and Asian counterparts (Colditz & Roster). In contrast to the United States with an incidence rate of 101/100,000, East Asian women have the lowest incidence rate of breast cancer at 21/100,000 (Ferlay et al; 2010). Even in Africa, this regional variation still exists, with studies showing that 27% of breast cancers was localized in women from the Northern part of Africa like Egypt, while 15% was reported in Sub Sahara region of Africa (Ganesh et al; 2010). In central Africa, incidence rate is 27/100,000 and 39/100,000 in the Southern part of Africa and that of China is 25.9/100,000 (Homes & Willett; 2004).

Breast cancer cases are gradually increasing in African countries, probably as a result of gradual adoption of more westernized lifestyle (Colditz & Roster; 2000). It is believed that breast cancer cases in Africa (Nigeria inclusive) will continue to increase as a result of ageing, increase growth of the population, people adopting unhealthy lifestyles, and absence of health related interventions against breast cancers (Kelsey & Bernstein; 1996). Late presentation, poor screening and diagnostic modalities, unavailability of quality health care infrastructures, poor health practices and funding, among other factors, are some of the factors that could lead to poor prognosis, increased morbidity and a low survival outcome in Nigerian women (Kay et al; 2001) and (Killias et al; 1999). It is important to note that not all tumors are cancers, hence there are benign and malignant tumors; cancers are malignant tumors. As a result of the resemblance that exists between cancer cells and normal cells from which they originate, they are not often detected by the immune system, especially if it is weakened<sup>11</sup>. Cancer cells are usually formed as a result of mutations of DNA or RNA. These mutations may occur spontaneously or may follow chronic exposures to nuclear radiation, electromagnetic radiation (X-rays, Gamma rays), viruses, bacteria, parasites, etc. It is thought that invasive cancers arise from series of molecular alterations at the cellular level. It is this alteration that results in breast epithelial cells that assume immortality and uncontrolled growth. There are other risk factors which can make a person more susceptible to developing breast cancer. Increasing age and female sex are known risk factors for the development of breast cancer. Consequently, breast cancer is 100 times more common in women than in men; however, men tend to have poorer prognosis mainly as a result of delay in diagnosis (Colditz & Roster; 2000). A positive family history is another known risk factor, including mutation carriers of BRCA 1 and 2.

Reproductive and hormonal factor such as late age at first pregnancy, early menarche, late age at cessation of menses, null parity have all been shown to be risk factors for the emergence of breast cancer (Pace & Shulman 2016) and (Page & Jensen; 1994). This is not unrelated to the effects of increased and unopposed estrogen in these women (Posner & Wolmark; 1992). Also, a previous history of breast cancer in one breast is associated with a 3-4 fold increase in occurrence of cancer in the other breast (Porter 2008) and (World Health Organization; 2020). Protective life styles against breast cancer can include consumption of diet rich in grains, fruits and vegetables, reduced saturated fats and low in alcohol (Sent et al; 2014). Early breast cancers are usually asymptomatic, and pain is usually not present at first.

Breast cancers may present as a lump with breast asymmetry, skin dimpling/ changes around the skin of the breast, nipple deviation or related changes, nipple discharge which could be bloody, axillary lump. Diagnosis and screening for breast cancer could involve any of the following or a combination; -physical examination of both breasts, mammography, ultrasonography, magnetic resonance imaging, biopsy. Management of breast cancers could include any of or a combination of the following – surgery, radiation therapy, hormone therapy, chemotherapy. This paper is aimed at ascertaining the age bracket of women that are susceptible to breast cancer, obtaining the state transition probabilities of breast cancer, estimating the ages that would be mostly affected with breast cancer in the future using Markov model and determining whether there are yearly increase in the number of breast cancer patients. The paper is organized as follows: Section one presents the methodology, model formation is seen in section 2.1, Result and Discussion are presented in section 3 and Conclusion in section 4.

**Methodology**

The study employed the Markov processes for stochastic modeling of breast cancer. Markov chain is a stochastic model that describes a sequence of possible events in which the probability of each event depends only on the state attained in the previous event. Thus if the current state of a process is known, its past states need not be required to predict its future state. This invariably means that a process’s past and future states are independent of its present state. One usefulness of this property of Markov model is that some parameters are likely to be redundant. Markov processes are widely used in different fields to model systems that can be in one or more states with certain probabilities. These probabilities are subject to changes over time. Transition matrix and state transition probabilities were developed in the case of this study. The transition matrix is usually given the symbol  $P = (P_{ij})$ .

In the transition matrix P:

The ROWS represent NOW or FROM ( $X_t$ )

The columns represent NEXT or TO ( $X_{t+1}$ )

The entry  $(i, j)$  is the conditional probability that NEXT =  $j$ , given that NOW =  $i$ ; the probability of going FROM state  $i$  TO state  $j$  is ;

$$P_{ij} = P[X_{t+1} = j | X_t = i]$$

The transition matrix P must list all possible states in the state space S. P is a square matrix ( $N \times N$ )

Because both  $X_{t+1}$  and  $X_t$  take values in the state space S (of size N)

The rows of P should sum to one (1).

$$\sum_{j=1}^N P_{ij} = \sum_{j=1}^N P(X_{t+1} = j | X_t = i)$$

$$\sum_{j=1}^N P\{X_t = i\} P(X_{t+1} = j) = 1$$

This simply states that  $X_{t+1}$  must take one of the listed values.

**The columns of P do not in general sum up to one (1)**

For the purpose of this study, the raw data was transformed into  $3 \times 3$  transition probability matrix. Thus three states were considered. A random walk is said to exhibit the Markov property if the position of the movement at time  $(n+1)$  depends only upon the position of the movement at  $n$ . Let  $X_n$  denote the position of the random movement at time  $n$ , then the equation becomes  $P(X_{n+1} = j | X_n = i) = P_{ij}$ .....3.1

Where  $P_{ij}$  is independent of  $X_{n-1}, X_{n-2}, X_{n-3}, \dots, X_0$  so that the state  $X$  at time  $(n + 1)$  depends only upon the state of  $X$  at time  $n$ .

This implies that each  $P_{ij}$  for  $j = 1, 2, 3, \dots, N$  is a probability row vector describing every possible transition from state  $I$  to any other existing  $N$  possible states in the process.

Thus for every  $I, \sum NP(ij = 1) \dots \dots \dots 3.2$

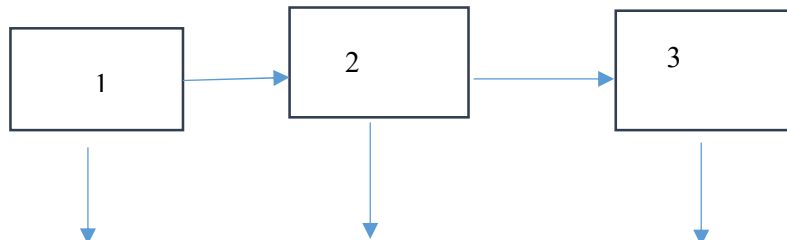
Generally, a random movement exists in  $N$  possible states in the system hence  $P(X_{n+1} = j)$  will depend on the whole sequence of random variables starting with the initial value  $X_0$  (Jones and Smith, 2001). This yields

$$P(X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, X_0 = i_0) = P(X_{n+1} = j | X_n = i) \dots \dots \dots 3.3$$

**Model formation**

In this research, the ages of breast cancer patients were grouped into stages (states). Thus we considered 0-44 (state 1) 45-58 (state 2) and 58 & above (state 3).

The model is presented below;



The horizontal arrow signifies transition from one state to the other, while the vertical arrow signifies exiting or leaving the system (death).

**Result and discussion**

The collected data were transformed to a 3 x 3 matrix called the transition state matrix and subsequently probability transition matrix represented by  $M$  and  $P$  respectively.

$$M = \begin{pmatrix} 23 & 87 & 45 \\ 22 & 133 & 60 \\ 21 & 124 & 53 \end{pmatrix}$$

$$p^0 = \begin{pmatrix} 0.1484 & 0.5613 & 0.2903 \\ 0.1023 & 0.6186 & 0.2791 \\ 0.1061 & 0.6263 & 0.2677 \end{pmatrix}$$

$$p^6 = \begin{pmatrix} 0.1084 & 0.6146 & 0.2772 \\ 0.1084 & 0.6146 & 0.2772 \\ 0.1084 & 0.6147 & 0.2772 \end{pmatrix}$$

$$P^{24} = \begin{pmatrix} 0.0929 & 0.5265 & 0.2315 \\ 0.0917 & 0.5201 & 0.2287 \\ 0.0928 & 0.5261 & 0.2313 \end{pmatrix}$$

We generated a simultaneous equation to determine the value of  $\pi$  which represents the state that would be susceptible to breast cancer in the future. Hence linear equations representing the limiting probabilities will be generated. The equations will be of the form  $\pi = \pi P$

Hence  $(\pi_1, \pi_2, \pi_3) \begin{bmatrix} 0.0929 & 0.5265 & 0.2315 \\ 0.0917 & 0.5201 & 0.2287 \\ 0.0928 & 0.5261 & 0.2313 \end{bmatrix}$

This gave rise to

$$\pi_1 = 0.0929\pi_1 + 0.5265\pi_2 + 0.2315\pi_3 \dots\dots\dots (1)$$

$$\pi_2 = 0.0917\pi_1 + 0.5201\pi_2 + 0.2287\pi_3 \dots\dots\dots (2)$$

$$\pi_3 = 0.928\pi_1 + 0.5261\pi_2 + 0.2313\pi_3 \dots\dots\dots (3)$$

$$\pi_1 + \pi_2 + \pi_3 = 1 \dots\dots\dots (*)$$

From eqns. 1, 2 and 3 we have;

$$9071\pi_1 = 5265\pi_2 + 2315\pi_3 \dots\dots\dots (4)$$

$$4739\pi_2 + 928\pi_1 + 2313\pi_3 \dots\dots\dots (5)$$

$$7687\pi_3 = 928\pi_1 + 5261\pi_2 \dots\dots\dots (6)$$

Rearranging and subtracting eqn.(6) from eqn.(5)

$$4739\pi_2 = 928\pi_1 + 2313\pi_3 \dots\dots\dots (5)$$

$$-5261\pi_2 = 928\pi_1 - 7687\pi_3 \dots\dots\dots (6)$$

$$10000\pi_2 = 10000\pi_3$$

$$\therefore \pi_2 = \pi_3 \dots\dots\dots (7)$$

From eqn.(4) we have ;

$$9071\pi_1 = 5265\pi_3 + 2315\pi_3$$

$$9071\pi_1 = 7580\pi_3$$

$$\therefore \frac{7580}{9071}\pi_3$$

Recall that  $\pi_1 + \pi_2 + \pi_3 = 1 \dots\dots\dots (*)$

By replacement we have;

$$\frac{7580}{9071}\pi_3 + \pi_3 + \pi_3 = 1$$

Taking L.C.M, we have ;

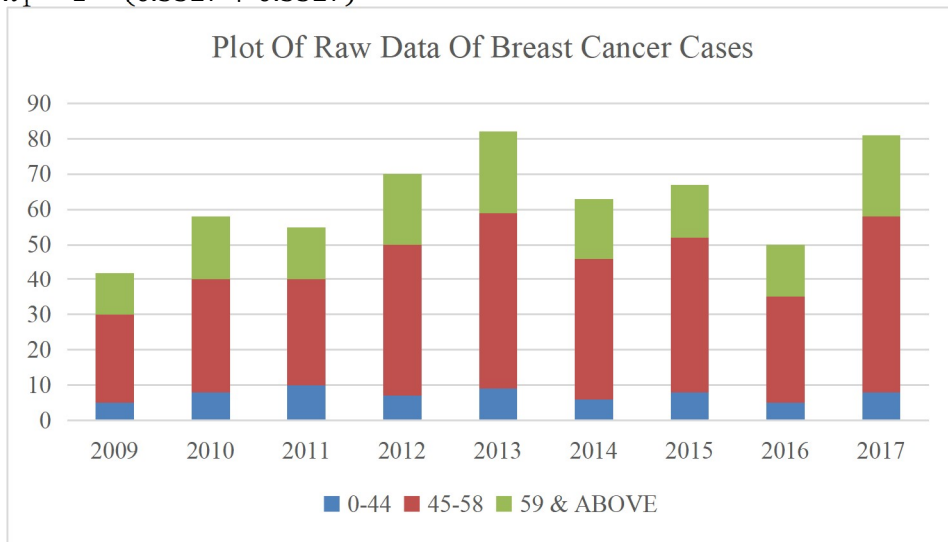
$$25722\pi_3 = 9071$$

$$\pi_3 = \frac{9071}{25722}$$

$$\therefore \pi_3 = 0.3527$$

From eqn.(7)  $\pi_2 = \pi_3$  hence  $\pi_2 = 0.3527$

$$\pi_1 = 1 - (0.3527 + 0.3527)$$



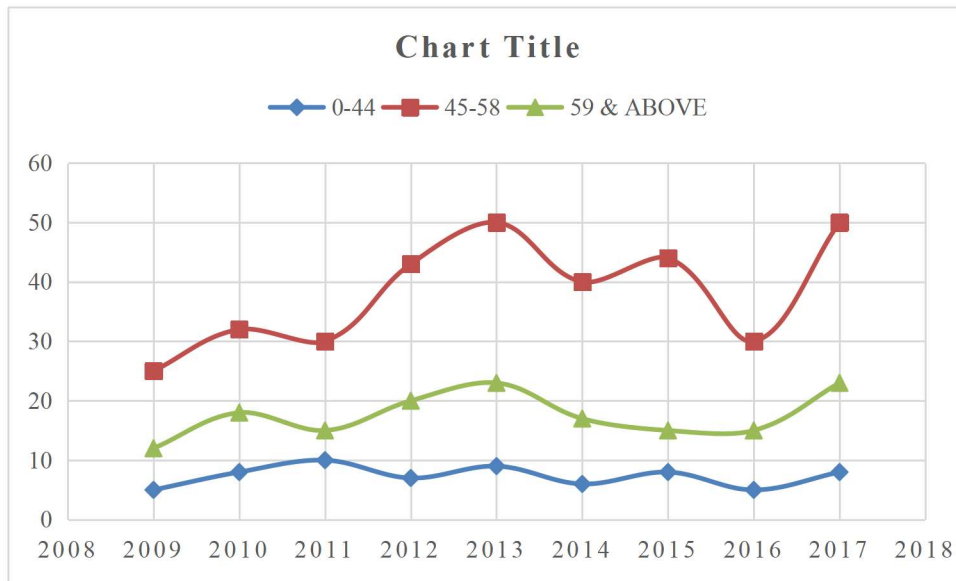
$$\pi_1 = 0.2946$$

From the values of  $\pi_1$ ,  $\pi_2$  and  $\pi_3$  we deduce that at the long run, the number of People with breast cancer at state 2 (45-57) will equal the number of People at state 3 (55 & above).

No. periods

Cancer cases

The graph above shows that breast cancer invasion increased yearly throughout the nine years of investigation. We also considered individual states of cancer against number periods (years). The plot agrees that the age 45-58 (state 2) recorded the highest breast prevalence.



### Summary of findings

We randomly considered a six year probability matrix in order to determine the state transition matrices of breast cancer prevalence in Rivers State. In the year 2024 , the number of breast cancer patient in state 2 will equal the number in state 3. This also indicates yearly increase.

### Conclusion

Breast cancer is the most common cancer in women and also the leading cause of death among women. Breast cancer invasion increased yearly throughout the nine years. Women between the ages of 45-58 suffered most from breast cancer.

### Recommendations

We recommend protective life styles against breast cancer such as consumption of diet rich in grains, fruits and vegetables, reduced saturated fats and low in alcohol. Also, women should be guided on the choice of contraceptives, clamping down on contraceptives that pose higher risks for occurrence of breast cancer like the oral contraceptive pills. Also, early detection and screening is highly recommended through less expensive modalities like physical examination, mammography and ultrasonography.

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