Interrupted Time Series Analysis Of South Africa Rand (Zar)/ Nigeria Naira (NGA) Exchange Rate

Wobo Omezuruike Gideon & Nwaigwe Echendu.
Deparment of Statistics,
Port Harcourt Polytechnic Rumuola, Port Harcourt
Omezuruike.wobo@portharcourtpoly.edu.ng, echendunwaigwe@yahoo.com

Abstract
The plot of the daily South-Africa rand (ZAR)/ Nigeria naira (NGN) exchange rate showed a downward trend and later upward. An intervention point was noticed from 17th Dec, 2017. The pre-intervention series is found to be non-stationary, and became stationary after the first difference. The Augmented Dickey-Fuller (ADF) test statistics result of -7.279482 is less than the critical values of -3.548208,-2.912631 and -2.594027, the p-value (0.0000) is less than 0.05 which implies stationarity. The ACF and PACF of the differenced series is found to be stationary, hence ARIMA (0, 1, 0) is fitted which is a simple random walk. Forecast was made on the basis of the stationarity of the differenced series and a comparison of the intervention forecast and post-intervention rates revealed strong agreement between both series, hence the overall intervention model $X_t = \frac{\varepsilon_t}{1-B} + 3.67531819\left(1 - 0.947528^{t-60}\right)I_t$ *is found to be adequate. This is clearly seen from the R-squared and adjusted R-squared which is approximately 77%.*
Keywords: *Exchange rate, Arima intervention model, Forecasting*

Introduction
There is a popular saying that the world is a global village. The internet affords people and various countries the opportunity to exchange goods and services which have great impact on the economy of each country. One of the most popular segments of the world's business is exchange rate, which is the value of one country's currency in relation to another currency (O'Sullivan and Steven, 2003). Forecasts are essential in both short-term and long-term plans. However, making such forecasts implies developing a model showing how the business world operates. Time series analysis has widely been applied in various fields of study since the publication by Box and Jenkins (1970, 1976). various research papers have become available and applied to vast real life problems such as: Etuk et al (2017), Herbury et al (2013), Amadi and Aboko (2013), Dobre et al (2008), Abbas and Allan(2004), Etuk et al (2017), Jerffrey and Eric (2011),Nihan et al (2005), Ismail et al (2009), Abdalla (2006), Yakubu et'al (2013), Nelson (2000), Zheng et al (2015), Krishnamurthi et al (1986), Yin and Newman (1999), Yang (2004) Onasanya and Adenyi (2013) Sharma and Khare (1999), Chang and Lin (1997),. Hilson (1984), Noland et al (2008), Amadi and Etuk (2017), Sastri and Valdes (1989). We will consider the exchange rate between South Africa rand and Nigeria naira using the intervention model approach proposed by Box and Taio (1975)

Materials and methods
The data for this research work is obtained from the website of the central bank of Nigeria (CBN) www.cbn.gov.ng/exrate.asp from the 16th day of October, 2017 to the 13th day of April, 2018. This research work will make use of time series Arima-Intervention approach to analyze the dynamics of the changes, variations and the interruption in the naira-rand exchange rate.

Arima-intervention model.

The ARIMA-Intervention model was introduced by box and Tiao (1975). The ARIMA-intervention method is used to ascertain the impact of the intervention on a time series. A simple intervention model with a single spike in the series is given by:

$$X_t = W_t + N_t \quad (1)$$

Where W_t is the intervention effect modeled as a transfer function that makes use of the pulse and step function to access the degree of the intervention, N_t is the pre-intervention process which is as an ARIMA model derived from box-Jenkins approach. W_t is assumed to be zero before intervention.

Suppose that X_t follows an ARIMA given by:

$$X_t - \mu = \frac{\Phi(B)\epsilon_t}{\Pi(B)} \quad (2)$$

And at time t=T there is intervention to X_t which changes the mean level of X_t, assuming the ARIMA structure of X_t in (2) holds before and after the intervention.

Let W_t be the magnitude of the change at time t=T, and let

$I_t = \begin{cases} 1 & t \geq T \\ 0 & t < T \end{cases}$ Then at time t=T, the overall model becomes:

$$X_t = W_t + \mu + \frac{\Phi(B)}{\Pi(B)}\varepsilon_t \quad (3)$$

The following pattern and model could be followed by W_t at time $t \geq T$,
That is the intervention period.

i) **Indefinite constant change at the mean level**

In this case there is a change either upward or downward to each value of the series after T. Let I_t be an indicator variable defined by:

$I_t = \begin{cases} 1 & t \geq T \\ 0 & t < T \end{cases}$ then W_t can be modeled as $W_t = \beta_o I_t$

Hence $W_t = \begin{cases} \beta_o & t \geq T \\ 0 & t < T \end{cases}$ β_o is estimated from the data. The pattern is shown in figure1 below.

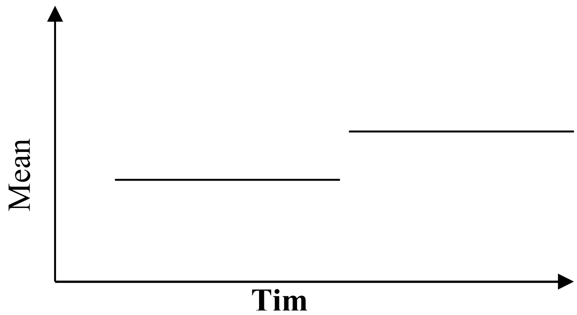


Figure 1: Indefinitely Constant

ii) Transient constant change for a period d times past T

In this case, the change is temporary lasting d times past the intervention period T. Define W_t by:

$W_t = \beta_o(1 - B^d)I_t$. Or we can redefine I_t to be $I_t = \begin{cases} 1 & T \leq t \leq T + d \\ 0 & for\ all\ other\ t \end{cases}$

And $W_t = \beta_o I_t$. Hence $W_t = \begin{cases} \beta_o & T \le t \le T + d \\ 0 & for\ all\ other\ t \end{cases}$

The pattern is given in the figure2 below.

Figure 2: Transient Constant Change for a period d

iii) A progressive increase that eventually becomes constant

In this case the change increases gradually and levels off. We define W_t

By: $W_t = \frac{\beta_o}{1-k_1 B} I_t = k_1 W_{t-1} + \beta_o I_t$

For $I_t = \begin{cases} 1 & t \ge T \\ 0 & t < T \end{cases}$ assume $|k_1|<1$, then W_t can be modeled as:

$$W_t = \begin{cases} \dfrac{\beta_o}{1-k_1} = k_1 W_{t-1} + \beta_o & t \ge T \\ 0 & t < T \end{cases}$$

W_t May be written in terms of β_o and k_1 as $W_t = \frac{\beta_o(1-k_1^{t-T+1})}{1-k_1}$

Where k_1 and β_o are estimated from the data. The pattern is shown in figure3 below.

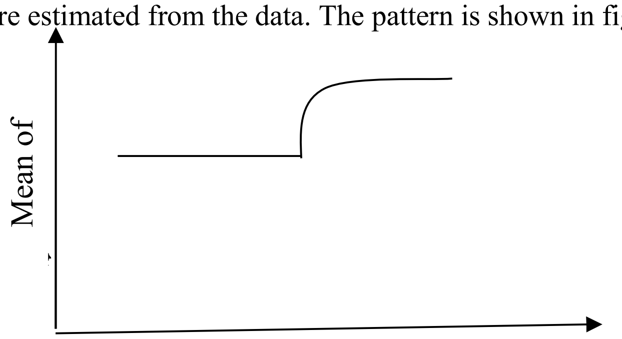


Figure 3: progressive increase that becomes

iv) Instant Change Which Eventually Returns to its original position.

We define $W_t = \frac{\beta_o}{1-k_1 B} p_t = k_1 W_{t-1} + \beta_o p_t$

The same condition above for k_1 holds. We define p_t by:

$p_{t=}\begin{cases}1 & t = T\\ 0 & otherwise\end{cases}$, then $W_t = \begin{cases}k_1 W_{t-1} & t \geq T+1\\ 0 & W_{t-1} = 0\ and\ t < T\\ \beta_o & W_{t-1} = 0\ and\ t = T\end{cases}$

The pattern is shown in figure3 below

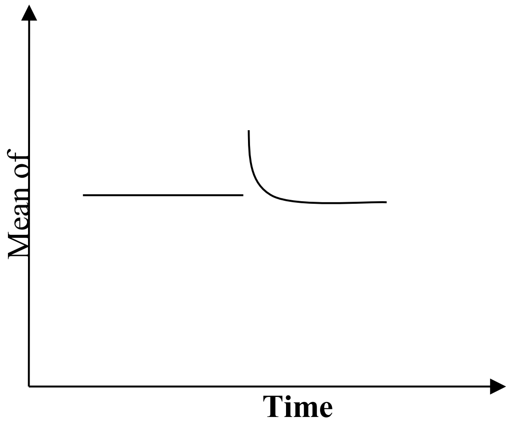


Figure 4: Instant change which returns to its initial position

We used Eviews 7 for all computations of this work.

Results and Discussion

The plot of the exchange rate in figure 1 rise to a certain level after 60 days and did not return, which is an intervention case. The plot of the pre-intervention exchange rate in figure 2 is an indication of a non-stationary series.
A stationarity test shown on table1 below reveal is in agreement with figure 1 for the t-statistics (-1.921472) is greater than the test critical values (-3.546099, -2.911730, -2.593551) and The probability 0.3205 is greater than 0.05. We differenced the series until it became stationary which is better confirmed from the plot of the differences of pre-intervention rates shown on figure3 below, the correlogram of the differenced series in figure 4 and the stationarity test on table 2. The t-statistic = -7.279482 is less than the critical values (-3.548208, -2.912631, -2.594027). also the probability is less than 0.05. as shown in table 2. The ACF and PACF shows that the difference of the pre-intervention rate is white noise (see figure 4), hence ARIMA(0,1,0) is fitted, that is

$$X_t = X_{t-1} + \varepsilon_t$$
$$= \frac{\varepsilon_t}{1-B} \tag{4}$$

we have the basis for forecasting the post-interventon period. Let F_t be the forecasts, then $W_t = X_t - F_t$. The graph is shown on figure5 below and the estimation of the transfer function on table 3.

The transfer function is thus given by:

$$W_t = \frac{\beta_o\left(1 - k_1^{t-60}\right)}{1 - k_1} \tag{5}$$

Where $\beta_o = \mathrm{C}(1)$ and $k_1 = \mathrm{C}(2)$. Substituting β_o and k_1 into (5) where $\beta_o = 0.192873$ and $k_1 = 0.947528$ yeilds

$W_t = \frac{0.192873(1-0.947528^{t-60})}{1-0.947528}$. Simplifying we have that

$$W_t = 3.67531819\left(1 - 0.947528^{t-60}\right) \tag{6}$$

Combining (4) and (6), we have the overall intervention model to be

$$X_t = \frac{\varepsilon_t}{1-B} + 3.67531819\left(1 - 0.947528^{t-60}\right)I_t \tag{7}$$

Where $I_{t=} \begin{cases} 1 & t \geq T \\ 0 & otherwise \end{cases}$

Conclusion

An Arima-intervention model is fitted to South Africa Rand/Nigeria Naira exchange rate data. The R-square and adjusted R-square which is 77% shows that the model is a good fit for the data. The pattern for the intervention is a progressive increase that eventually becomes constant. The change increased gradually and levels off which is not favorable to Nigeria’s economy as the change did not return to its original position.

APPENDIX

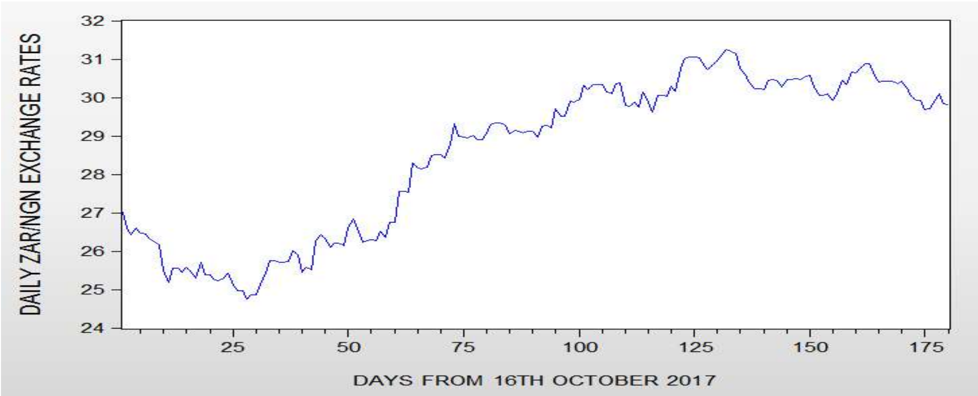


Figure 1: **Time Plot of the daily ZAR /NGN exchange rates**

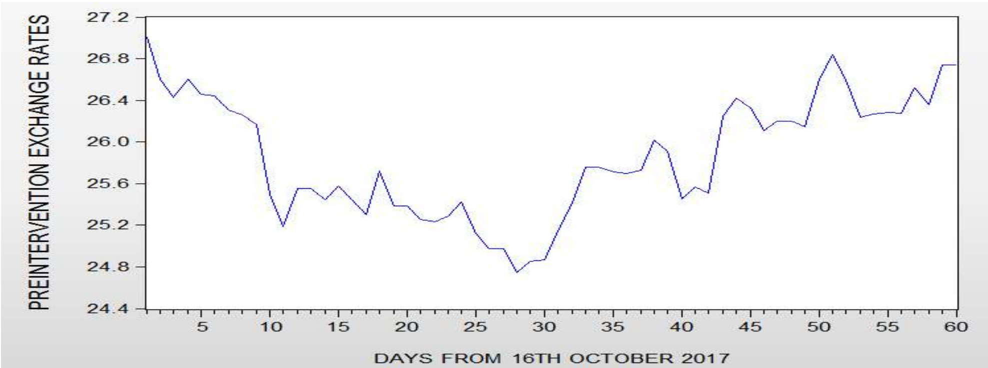


Figure 2: Time Plot of the pre-intervention exchange rates

Table 1: Stationarity test for the pre-intervention series

Null Hypothesis: ZARN has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-1.921472	0.3205
Test critical values:	1% level	-3.546099	
	5% level	-2.911730	
	10% level	-2.593551	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(ZARN)
Method: Least Squares
Date: 04/18/18 Time: 11:21
Sample (adjusted): 2 60
Included observations: 59 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
ZARN(-1)	-0.104943	0.054616	-1.921472	0.0597
C	2.708171	1.412110	1.917819	0.0601

R-squared	0.060833	Mean dependent var	-0.004508
Adjusted R-squared	0.044356	S.D. dependent var	0.242935
S.E. of regression	0.237486	Akaike info criterion	-0.004105
Sum squared resid	3.214780	Schwarz criterion	0.066320
Log likelihood	2.121093	Hannan-Quinn criter.	0.023386

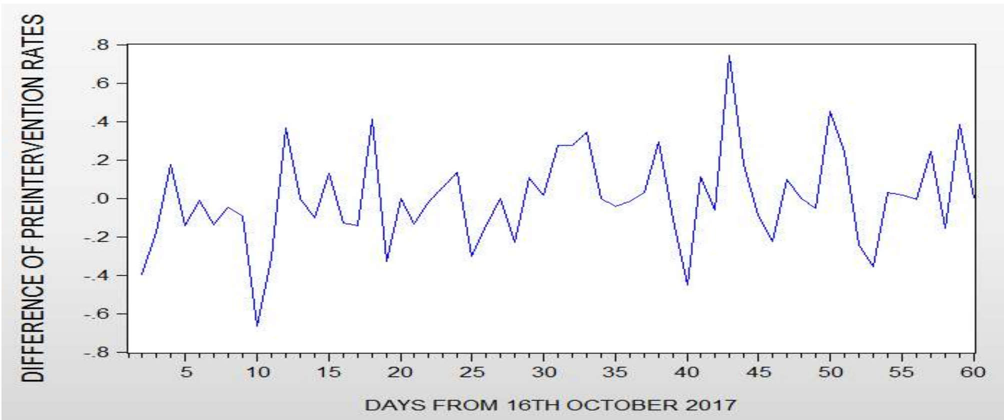


Figure 3: Differences of the pre-intervention rates

Table 2: Stationarity test for the differences of the pre-intervention series

Null Hypothesis: DZARN has a unit root
Exogenous: Constant
Lag Length: 0 (Automatic - based on SIC, maxlag=10)

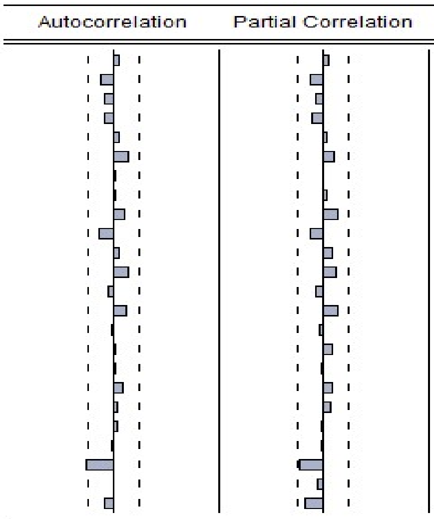
		t-Statistic	Prob.*
Augmented Dickey-Fuller test statistic		-7.279482	0.0000
Test critical values:	1% level	-3.548208	
	5% level	-2.912631	
	10% level	-2.594027	

*MacKinnon (1996) one-sided p-values.

Augmented Dickey-Fuller Test Equation
Dependent Variable: D(DZARN)
Method: Least Squares
Date: 04/18/18 Time: 11:23
Sample (adjusted): 3 60
Included observations: 58 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
DZARN(-1)	-0.948591	0.130310	-7.279482	0.0000
C	0.002541	0.031662	0.080238	0.9363

R-squared	0.486195	Mean dependent var	0.006852
Adjusted R-squared	0.477020	S.D. dependent var	0.333380
S.E. of regression	0.241092	Akaike info criterion	0.026596
Sum squared resid	3.255015	Schwarz criterion	0.097646
Log likelihood	1.228706	Hannan-Quinn criter.	0.054272



	AC	PAC	Q-Stat	Prob
1	0.051	0.051	0.1640	0.686
2	-0.116	-0.119	1.0168	0.601
3	-0.083	-0.071	1.4575	0.692
4	-0.091	-0.099	1.9977	0.736
5	0.044	0.036	2.1284	0.831
6	0.143	0.114	3.5074	0.743
7	0.023	0.008	3.5432	0.831
8	0.012	0.039	3.5541	0.895
9	0.104	0.136	4.3397	0.888
10	-0.148	-0.137	5.9484	0.820
11	0.054	0.098	6.1688	0.862
12	0.155	0.128	7.9997	0.785
13	-0.064	-0.079	8.3226	0.822
14	0.125	0.155	9.5747	0.793
15	-0.015	-0.039	9.5938	0.844
16	0.022	0.097	9.6331	0.885
17	0.016	-0.011	9.6544	0.917
18	0.093	0.089	10.419	0.917
19	0.034	0.077	10.522	0.939
20	0.038	-0.014	10.657	0.955
21	-0.022	-0.008	10.702	0.968
22	-0.270	-0.245	17.780	0.719
23	-0.004	-0.055	17.781	0.769
24	-0.082	-0.173	18.480	0.779

Figure 4: Correlogram of differences of the pre-intervention series

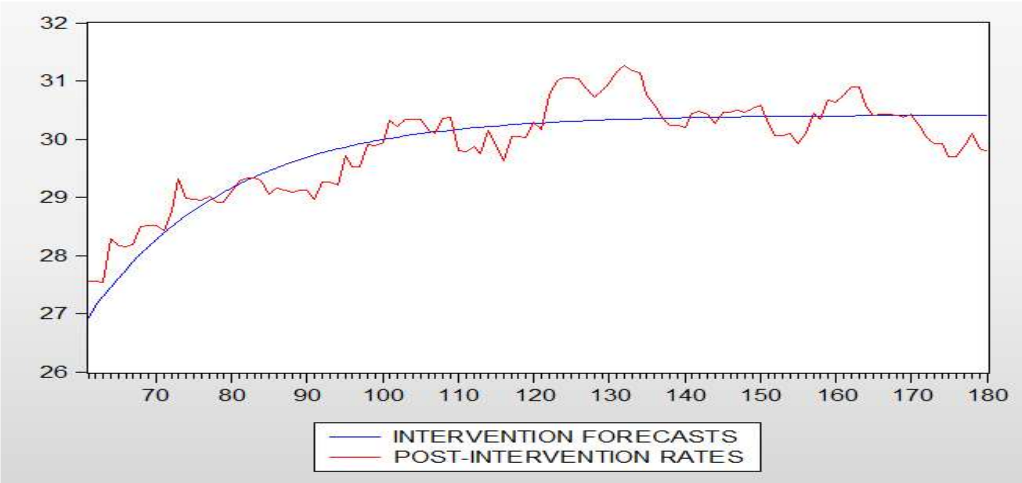


Figure 5: Comparison of the intervention forecasts and post-intervention rates

Table 3: Estimation of the intervention transfer function

Dependent Variable: Z
Method: Least Squares (Gauss-Newton / Marquardt steps)
Date: 04/18/18 Time: 12:08
Sample: 61 180
Included observations: 120
Convergence achieved after 14 iterations
Coefficient covariance computed using outer product of gradients
Z=C(1)*(1-C(2)^(T-60))/(1-C(2))

	Coefficient	Std. Error	t-Statistic	Prob.
C(1)	0.192873	0.010937	17.63560	0.0000
C(2)	0.947528	0.003438	275.6264	0.0000

R-squared	0.768048	Mean dependent var	3.149546
Adjusted R-squared	0.766082	S.D. dependent var	0.826752
S.E. of regression	0.399859	Akaike info criterion	1.021118
Sum squared resid	18.86671	Schwarz criterion	1.067576
Log likelihood	-59.26705	Hannan-Quinn criter.	1.039984
Durbin-Watson stat	0.252915		

References

Abbas, V. & Allan, P. L. (2004). Quantifying the effect of the goods and services tax on inflation in Australia's capital cities: An intervention analysis. *The Australlian Economic Review, 37* (2), 125-138.

Abdalla, E. H. (2006). Measuring the impact of a new policy on industry in Gaza. *An-Naiah cenukeisity Journal Res. (H. Sc), 21*(2).

Amadi, I. U. & Aboko, S. I. (2013). The fitting of Arima model in forecasting Nigeria Gross Domestic Product. (GDP). *Journal of Physical Science and Innovation*, *5* (2), 79-90.

Box, G. E. P. & Tiao, G. C. (1975). Intervention analysis with application to economic and environmental problems, *JASA, 70,* 70-79.

Box. G. E. P, & Jenkins, G. M. (1976). *Time series analysis: Forecasting and ccontrol*, San Franscisco Holden-Day.

Chang, N. B. & Lin, Y. T. (1997). An analysis of recycling impact on solid waste generation by time series intervention modeling. *Resource Conservation Recycling 19*, 165-186.

Dorbe, I., Alexandru, A. & Ana, M. (2008). Modeling unemployment rate using Box-Jenkins procedure, *Journal of Quantitative Methods, 3*(2), 156-166

Etuk, E. H., Maxwell, A. A. & Elenga, N. B. (2017). Interrupted time series of daily amount of British pound per Euro due to Braxit. *Journal of Science and Engineering Research, 4*(10), 233-237.

Etuk, E. H., Nnoka, L. C. & Etukudo, I. A. (2017). Arima intervention of daily west African france (X0F) Nigeria naira (NGN) exchange rates. *International Journal of Advanced Research in Computer Science and Software Engineering, 7* (5).

Etuk, E. H, Agbam, A. & Moffat, I. (2017). Interrupted time Series Analysis of Nigerian Monthly Distribution of Petroleum Products. *World Wide Journal of Multidisciplinary Research and Development. 3*(7), 108-113.

Hanbury, A., Farley, K., Thompson, C. Willson, P. M., Chambers, D. & Holmes, H. (2013). Immediate versus sustained effects: interrupted time series analysis of a tailored intervention Implementation science. 2013 Nov 5, 8:130. dio:10.1186/1748-5908-8-130.

Hilton, M. E. (1984). The impact of recent changes in califonia's drinking-driving laws on fatal accident levels during the first post intervention year. An interrupted time series analysis. *Law and Society Review 18*(4), 605-627.

Ismail, Z. S, Yahaya, A. & Efendi, R. (2009). Intervention model for analysis the impact of terrorism to tourism industry. *Journal of Mathematics and Statistics 5*(4), 322-329.

Jerfrey, E. J. & Eric, K. (2011). Arima modeling with intervention forecast and analysis of Chinese stock price. *International Journal of Engineering and Business Management, 3,* (3), 53-58.

Krishnamurthi, I., Narayan, J. & Raj, S. P. (1986). Intervention analysis of a field experiment to access the building effect of advertising. *Journal of Marketing Research, 27*, (4), 337-345.

Nelson, J. P. (2000). Consumer bankruptcies and the bankruptcy reform act. A time series intervention analysis. *Journal of Financial Series Research, 17*(2), 181-200

Noland, R. B. M. A., Qddles, W. T. & Ocheng, (2008). The effect of the London congestion change on road casualties. An intervention analysis. *Transportation 35*, 73-91.

O'sullivan, A. & Steven, M. S. (2003). Economic principle in action. Upper saddle rivers, New Jersey 07458: Pearson prentice hall p.458. ISBN 0-13-06 3085-3.

Onasanya, O. K. & Adeniyi, O. E. (2013). Forecasting for exchange rate between Naira and US dollar using time domain model. *International Journal of Development and Economic Sustainability, 1*(1), 45-55.

Sastri, T. & Valdes, I. B. (1989). Rainfall intervention analysis for online Applications. *Journal of Water Resources, Planning and Management, 115* (4). *Doi:http:/dx.Doi.org/10./061/(ASACE) 0733-9496 (1989) 115:49(397)*

Sharma, P. & Khare, P. (1999). Application of intervention analysis for assessing the effectiveness of CO pollution control legislation in India Transformation Res. *Part, 4*, 427-432.

The Pennsylvania state University (20160. Welcome to STAT 510. Applied Time series analysis, Department of Statistics online Program. www.onlinecourses.science.psu.edu/stst510/

Yakubu, M. Y., Shehu, L. & Mukhtar, G. (2013). Modelling and forecasting of Nigeria crude oil prices using Box-Jenkins technique. *Journal of Physical Science and Innovation*, *5* (2), 50-59.

Yin, R. & Newmann, D. H. (1999). An intervention analysis of hurricane hugo's effect on south carolina's stumpage prices. *Canadian Journal of Forest Research, 2* (6), 179-189.

Zheng, T., Farrish, J. & Kitterlin, M. (2015). Performance trend of hotels and casino hotels through recession: An arima with intervention analysis of stock indices. *Journal of Hospitality Marketing, 25*(1), 49-68.